In the interpretations of numerical expressions, the relevant set will be the counting base set in the lexical entries of nouns. This gives us the right selectional restrictions for numerical expressions, namely that they can only felicitously combine with count nouns (barring coercion).

We assume that numerical expressions in English and German are of the equivalent of an adjectival (modifier) type (the equivalent in this system of type $\langle\langle s,\langle et\rangle\rangle,\langle s,\langle et\rangle\rangle\rangle$). This is represented in (46). Where $\mathcal{P}$ is a variable over common noun interpretations such as $[\![$information$]\!]$, $[\![$Information(en)$]\!]$ ('piece(s) of information', German), $[\![$kitchenware$]\!]$ and $[\![$Küchengerät-e$]\!]$ ('piece-s of Kitchenware', German). This means that we must assume that $\mathcal{P}$ is of an underspecified type, i.e., underspecified between the type for entity denoting noun interpretations and proposition denoting noun interpretations. The variables $\chi, \mathfrak{y}$ are underspecified between type e and type $\langle s,t\rangle$. The schema for a noun lexical entry is given in (43).

(43) $$\lambda w.\lambda\chi.\begin{bmatrix} ext & = & P_w(\chi) \\ cbase & = & \lambda\mathfrak{y}.P_w(\mathfrak{y}) \end{bmatrix}$$

In addition, we use two projection functions *CBASE* and *EXT* such that:

(44) $$\lambda w.\lambda\chi.CBASE(43)(\chi)(w) = \lambda\mathfrak{y}.P_w(\mathfrak{y})$$

(45) $$\lambda w.\lambda\chi.EXT(43)(\chi)(w) = P_w(\chi)$$

The interpretation of *two* (or equivalently *zwei* in German) can now be stated as follows in (46). This is a function that applies to a common noun and returns an entry with the same counting base set and with an extension that is restricted to only include entities with a cardinality of 2 with respect to the counting base set.

(46) $$[\![[_{Num}\text{ two}]\!] = [\![[_{Num}\text{ zwei}]\!] =$$
$$\lambda\mathcal{P}\lambda w.\lambda\chi.\begin{bmatrix} ext & = & EXT(\mathcal{P}(\chi)(w)) \wedge \mu_{\#}(\chi, CBASE(\mathcal{P}(\chi)(w))) = 2 \\ cbase & = & CBASE(\mathcal{P}(\chi)(w)) \end{bmatrix}$$

Counting constructions

This semantics for numerical expressions, i.e. modifiers that restrict the extension of a noun to entities that have a cardinality of n with respect to the counting base set, automatically selects for count nouns, given the definition of $\mu_{\#}$. This means that we can straightforwardly account for the interpretation and felicity of counting constructions with both concrete count nouns and count IONs such as *zwei Küchengeräte* ('two pieces of Kitchenware', German) and *zwei Informationen* ('two pieces of information', German), respectively. The derivations for these are based on (46) and the entries for *Küchengeräte* (28) and *Informationen* (33) in section 3

(47) $$[\![[_{NumP}\text{ zwei Küchengeräte}]\!]^c = [\![[_{Num}\text{ zwei}]\!]([\![[_N\text{ Küchengeräte}]\!]^c) =$$
$$\lambda w.\lambda x.\begin{bmatrix} ext & = & {}^*\mathcal{Q}_c(kitchenware_w)(x) \wedge \mu_{\#}(x, \mathcal{Q}_c(kitchenware_w)) = 2 \\ cbase & = & \lambda y.\mathcal{Q}_c(kitchenware_w)(y) \end{bmatrix}$$

(48) $$[\![[_{NumP}\text{ zwei Informationen}]\!]^c = [\![[_{Num}\text{ zwei}]\!]([\![[_N\text{ Informationen}]\!]^c) =$$
$$\lambda w.\lambda p.\begin{bmatrix} ext & = & {}^*\mathcal{Q}_c(information_w)(p) \wedge \mu_{\#}(x, \mathcal{Q}_c(information_w)) = 2 \\ cbase & = & \lambda q.\mathcal{Q}_c(information_w)(q) \end{bmatrix}$$

For the English mass CAN and mass ION cases (*#two kitchenwares/informations*), composition with $[\![[_{Num}$ two$]\!]$ is ruled out since the counting base sets of $[\![[_N$ kitchenware$]]\!]$ and $[\![[_N$ information$]]\!]$ are not quantized, and so the use of these sets with $\mu_{\#}$ is undefined.