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9.1 Granular Nouns as a Notional Class

Granular noun is the label we use to refer to a semantic class of nouns denoting entities that consist of relatively small grains, particles or distinguishable pieces (e.g. *lentil, rice, pebble, sand, seed, barley, gravel*).^{*} These nouns form a disparate notional class that includes 'naturally' occurring objects (*rice*) and also artifacts (*sequin(s)*). Nouns in this class can be grammatically encoded as mass (e.g. *rice, barley, gravel*) or count (e.g. *pebble(s), lentil(s)*). Some languages also have dual life granular nouns (e.g. *seed*, cp. *many seeds, much seed*) and pluralia tantum granular nouns (*oats*).¹

When count, granular nouns are bona fide count nouns in that they are straightforwardly felicitous when directly modified by numerical expressions (*three pebbles/lentils*). However, this does not mean that they pattern distributionally like prototypical count nouns, such as *cat*. Many granular count nouns are, in terms of distributional frequencies, more often than not used in the plural (at least when they are the head of an NP, thus excluding non-granular-headed compounds, e.g. *lentil soup, bean burger*). Furthermore, in some contexts at least, plural granular nouns are slightly awkward to use with count quantifiers like *many*. For example, it is odd to ask when serving dinner, *How many lentils would you like*?, which suggests that we often do not care or do not focus on specific numbers.

When dual life, granulars such as *seed* are straightforwardly felicitous in both count and mass constructions. However, the mass sense seems to be restricted to collections of grains; hence, in mass constructions, dual life

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¹ Some native speakers find the direct numerical modification of *oat(s)* felicitous, however. For instance, some speakers accept *one oat* as straightforwardly felicitous, suggesting that for some native speakers of English, *oat* is a granular count noun akin to *lentil*.

granulars tend to resist any kind of 'grinding' reading which accesses the part structure of the individual grains. For instance, *After the silo explosion, there was seed all across the farmyard* does not seem to have an interpretation in which there are bits of individual seeds all over the farmyard. In this way, the count sense of dual life granular nouns is different from that of count nouns which do permit 'grinding', modulo context, as in, for example, *After the crash, there was motorbike strewn across the road*.

When mass, granulars are bona fide mass nouns. Also, as argued by Landman (Chapter 6 in this volume), the 'grains' in the denotation of granular nouns are not straightforwardly accessible for cardinality comparisons in comparative constructions. If a has two large grains of wild rice and b has three small grains of pudding rice that total less in volume, it is not obvious that 'b has more rice than a' has a true reading. Nonetheless, the fact that the denotations of granular mass nouns are made up of grains/granules is highly salient, even if this grain structure is inaccessible to the counting operation and to cardinality comparisons in comparative constructions.

Mass granular nouns will be the main focus of this paper. In particular, we introduce a puzzle relating to why expressions like *three rices* cannot be coerced to mean 'three grains of rice'. We label this puzzle *the accessibility puzzle*.

9.2 The Accessibility Puzzle

Concrete mass nouns can generally be coerced into count noun interpretations, such as CONTAINER, PORTION or SUBKIND, depending on context, a fact that has garnered much attention in formal semantics and philosophy since at least Pelletier (1975). (In this paper, we will not discuss the portion reading, however; see Landman, Chapter 6 in this volume, for extensive discussion of the portion reading.) Different classes of nouns diverge with respect to the ease with which they can be coerced into a count interpretation, however. For example, *water* in (1a)–(1b) is easier to coerce into a count noun interpretation than the granular mass noun *rice* (2a)–(2b), while count interpretations of *mud* are possible only in highly specialized contexts, such as technological ones, as illustrated in (3a)–(3b).

(1)	a.	Three waters, please!		
		e.g. three [GLASSES/BOTTLES OF] water.	(container or portion)	
	b.	I ordered three waters for the party: still, sparkling, and fruit-flavored for the kids.		
		i.e. three [KINDS OF] water	(subkind)	
(2)	a.	We ordered the main courses with two plain rinnan, more than enough for the four of us. ²	ce, one egg fried rice and a	
		e.g. two [BOWLS OF] plain rice	(container or portion)	

² www.derbytelegraph.co.uk/speciality-dishes-star-turn-littleover-s-red/story-20536589-detail/ story.html [accessed October 10, 2016].

b. CONTEXT: three kinds of rice: Calmati, Texmati, Kasmati
 These three rices have basmati's viscosity and cooking style, but smaller individual grains.³
 i.e. three [KINDS OF] rice (subkind)

 (3) CONTEXT: yield points of different mud samples before contamination The three muds experienced particle dispersion at the same temperature with different yield points.⁴
 a. e.g. The three [SAMPLES OF/VIALS OF] mud ... (container or portion)
 b. The three [KINDS OF] mud ... (subkind)

Chierchia (2010) poses the question, 'Why each time that we want to count using a mass noun don't we simply, automatically "apportion" it as needed? What is to prevent us from interpreting "water" as meaning something like "water amount" or "water quantity"?' His conclusion is that the mass/count distinction must be rooted in the grammar of natural languages, and not only in 'general cognition'. We agree with Chierchia, but, in addition, we bring to the table a complexity that raises a follow-up question. While it is true that concrete mass nouns can often be coerced into count noun interpretations given the right context, what has been less explored are cases in which mass-to-count shifts are prohibited or heavily restricted, despite cognitive and contextual factors that would prima facie seem to facilitate them. If the mass/count distinction is in the grammar, why are some coerced mass-to-count interpretations less accessible than others?

Let us first consider so-called **object mass nouns**, also known as **fake mass nouns**, which include *furniture*, *footwear*, *cutlery*, *crockery* and *equipment*, among many others. They have played a key role in the development of many recent theories of the mass/count distinction (Chierchia 1998; Barner and Snedeker 2005; Landman 2011, 2016; Sutton and Filip 2016a, 2016b, 2018). As the examples below show, they strongly resist coercion in numerical counting constructions in which either OBJECT UNITS (4), basic-level KIND UNITS (5a) or superordinate level KIND UNITS (5b) are counted:

(4) #I ordered three furnitures from Ikea: one table and two chairs.

(5) a. #I ordered two furnitures from Ikea: chairs and tables.b. #I ordered two furnitures from Ikea: bedroom and living room furniture.

³ Hensperger, Beth, and Julie Kaufmann (2003). *The Ultimate Rice Cooker Cookbook*, p. 23. Boston, MA: Harvard Common Press.

⁴ Adekomaya, Olufemi A. (2013). Experimental analysis of the effect of magnesium saltwater influx on the behaviour of drilling fluids. *Journal of Petroleum Exploration and Production Technology* 3: 61–67.

At face value, it seems puzzling that counting of object units in the denotation of *furniture* is prohibited, given that it consists of conceptually and visually salient individuated entities like individual chairs and tables, for instance. However, these are not accessible for grammatical counting, as the oddity of (5a) shows. Such data are well known, but rarely directly addressed. Rothstein (2015) highlights the fact that object mass nouns lack pluralization with a subkind interpretation (but offers no account). Landman (2011) criticizes the account of Chierchia (1998), in which object mass noun denotations are atomic, which would prima facie seem to predict that they should be accessible to direct grammatical counting, but nonetheless, they are not:

The problem is that it is not particularly difficult to semantically or contextually pull a set of atoms out of an atomic structure ... a child can do it. And there, of course, is the problem: the child doesn't do it. (Landman 2011)

This, among other considerations, motivated Landman to develop a theory that characterizes object mass nouns (his 'neat mass nouns') as those mass nouns expressing a concept which *overdetermines* what counts as one item for counting. In brief, the reason why we do not 'pull a set of atoms out of an atomic structure' is that there are many different ways of partitioning the domain into sets of entities each of which is 'one' for the purposes of counting, none of which is privileged over others, and such alternative partitions have members that overlap. Therefore, there is no single determinate way of counting, but rather multiple alternative ones in any given situation.

Building upon the ideas of Landman, in Sutton and Filip (2016b), we directly address restrictions on grammatical counting of object mass nouns that concern the cardinality of ordinary particular objects, which instantiate basic-level kinds, as in (4). In Sutton and Filip (2018), we address restrictions on grammatical counting of subkinds of object mass nouns (see also Grimm and Levin 2017). In both cases, we argue that the restrictions can be derived from *overlap*. In simple terms, object mass nouns overdetermine what counts as one particular unit, and also as what counts as one subkind. This overdetermination leads to overlap, and overlap blocks access to a countable set of object units or to a countable enumeration of subkinds. The use of object mass nouns in counting constructions requires an explicit unit extracting expression (e.g. *item of*) or an explicit kind extracting expression (e.g. *kind of*).

Now, it turns out, as we observe, that some of the countability properties of object mass nouns are shared by granular mass nouns. Both impose one specific restriction on the range of their admissible mass-to-count shifts: namely, while they ALLOW shifts to portions and subkinds (2a)–(2b), they strongly resist shifts to ordinary particular OBJECT UNITS (e.g. single grains), when they are, for instance, directly modified by numericals (6a)–(6b) or combined with a distributive determiner, as in sentence (7):

- (6) a. # Three rices fell off my spoon.
 b. # Drei Reis sind vom Löffel gefallen. (German) Three rice.sg be.3.AUX from.the.DAT spoon fall.PST-PTCP
- (7) # I removed each gravel from the sole of my boot.

This shared property of object mass nouns and granular mass nouns has not yet been noticed at all (to the best of our knowledge), which might be due to the fact that when it comes to the mass domain, granular mass nouns have received much less attention than object mass nouns (but see Chierchia 2010, Grimm 2012, Landman, Chapter 6 in this volume, Hnout 2017).

To sum up our observations thus far, mass nouns such as mud and blood are hard to coerce into container or portion readings, barring highly specialized contexts. With respect to mass-to-count object unit shifts, it is, of course, unsurprising that there is no mass-to-count object unit shift available for such prototypical mass nouns, because they denote substances lacking conceptually salient individuated units in their denotations. By the same token, it is, however, unclear why object (or fake) mass nouns and granular mass nouns do not sanction a shift into count interpretations that would involve what are conceptually, and possibly also functionally, salient and individuated OBJECT UNITS, in their denotations: for example, individual pieces or items of furniture like tables, stools, chairs, grains of rice and kernels of wheat. In order to use them felicitously in numerical counting constructions or with quantifiers that select for count predicates, it is not sufficient that we know what is (taken to be) a single countable object unit, such as individual pieces of furniture like chairs, grains of rice or kernels, but rather it is necessary that such nouns first be combined with an overt classifier-like 'object unit' expression, or a 'unit excerpting' operator (in the sense of Talmy 1986) that singles out "a single instance of the specified equivalent units" and sets them "in the foreground of attention" (Talmy 1986, p. 12), which can then serve as input into grammatical counting: for example, a piece/an item, as in a piece /an item of furniture, a grain, as in a grain of rice, or a blade, as in a blade of grass.

At first blush, then, it looks as though granular mass nouns pattern with object mass nouns in that the perceptually, cognitively or functionally determined units from which their denotations are built are not accessible for count reinterpretation via coercion. However, there are differences, too. While most would agree that there are minimal units in the denotations of both object mass nouns and granular nouns, with, for example, single chairs and tables as the minimal units in the denotation of *furniture*, and the single grains of rice as the minimal units in the denotation of *rice*, these entities that count as 'one' for counting for object mass nouns are not the *only* units that can be grammatically counted (Landman 2011). This is not the case for granulars, however.

- (8) Tan bought one item of furniture.
- (9) Tan ate one grain of rice.

The sentence in (8) can be true when Tan bought a stool, a dressing table and a mirror, which together form a single functional unit which we refer to as *a vanity*. Although single mirrors, stools and tables may be minimal in the denotation of furniture, the expression *one item of furniture* can also be used to refer to sums of such minimal entities. A parallel situation, however, does not hold for (9). One cannot use *one grain of rice* to refer to any sums of grains.

It is not entirely clear how granular nouns could be integrated into extant theories of the mass/count distinction. Let us take, as an example, Landman's (2011) overlap-based proposal, which has influenced much of our own work (Sutton and Filip 2016a, 2016b, 2017, 2018; Filip and Sutton 2017). On this view, as observed above, what makes object mass nouns mass is that the set of entities that count as one contains, in the same context, overlapping entities (e.g. a table and a vanity which subsumes that table). Granular mass nouns are different from object mass nouns, because for granular mass nouns the set of object units that intuitively count as one in their denotation is always disjoint. Nevertheless, such minimal granular object units are not accessible to grammatical counting operations, directly or via mass-to-count coercion. For instance, rice denotes non-overlapping grains, each clearly demarcated and disjoint from the other, and yet rice cannot be used in counting constructions that directly count individual grains of rice. In order to motivate this behavior, we cannot obviously rely on the overlap property which Landman (2011) compellingly uses to motivate why object mass nouns like *furniture* are not straightforwardly felicitous in grammatical counting construction. We will call the particular restriction on coercing object mass nouns and granular mass nouns to count interpretations that directly count particular OBJECT UNITS, as outlined above, the 'accessibility puzzle':

Why should conceptually and perceptually salient object units in the denotations of object mass nouns and granular mass nouns not be *directly* accessible by semantic counting operations, nor facilitate the mass-to-count *coercion*?

Specifically, when it comes to granular mass nouns, which are the main focus of this paper, we will address the following questions:

- (Q1) What is the semantic distinction between count granular nouns (*pebble* (*s*), *lentil*(*s*)) and mass granular nouns (*rice*, *barley*)?
- (Q2) Why do granular mass nouns (*rice*) strongly resist a coercion to count readings which involve the object units in their denotation (individual

grains of rice in the denotation of *rice*), even if the context makes such object units clearly conceptually and perceptually salient? For granular mass nouns, why are only shifts to count interpretations with implicit containers available, but not shifts to count interpretations based on individual units?

In relation to (Q1), we propose a characterization of the semantics for *basic* predicates for granular nouns which underspecify whether single grains or aggregates of grains are referred to. This representation captures our general knowledge (common across all granular nouns), that, inter alia, they are made up of grains, and typically come in clustered aggregates. Then, using the mechanisms for individuation that we have independently motivated elsewhere, we outline how such basic predicates can be mapped into a count interpretation (such as with the English lentil) or a mass interpretation (such as with the Czech čočka ['lentil', mass]). Such mappings rely, crucially, on two key ingredients: the *object identifying function*, which identifies perceptually or functionally salient entities in a noun's denotation, and the schema of individuation, which concerns a perspective on these entities relative to a context of utterance. Count granular noun lexical entries feature the object identifying function and a schema of individuation. Mass granular noun lexical entries lack the object identifying function and have a null schema of individuation.

In relation to (Q2), we argue that there is a key difference between mass-tocount coercion in which a contextually salient receptacle concept is used (when *three rices* can be used to mean 'three BOWLS OF rice', for instance), and mass-to-count coercion in which single, individuated object units are referred to (were *three rices* able to mean 'three GRAINS OF rice', for instance). In the former case, the implicit receptacle concept (e.g. BOWL) supplies a means of partitioning a variety of suitable domains (liquids, granulars and substances) into countable units, either as stuff contained in the receptacle or as stuff to the amount that could be contained within the receptacle. In contrast, mass-to-count unit shifts, were they possible, would amount to simply selecting salient entities in a mass noun's denotation and making them countable (and so, were they possible, would not, strictly, be coercion at all, given the standard view that coercion involves retrieving and using non-lexically specified information from the context). We argue that such a shift would amount to a generalized, mass-to-count shifting operation. The intriguing consequence of this proposal is that it predicts that for languages with a grammaticized lexical mass/count distinction, object unit shifts are excluded. (This will be discussed in detail in Section 9.5.4).

In Section 9.3, we lay out the basis for our formal analysis and also give some background on other relevant accounts of the mass/count distinction. In

Section 9.5, we give an analysis of object unit extracting classifiers such as *grain of* and classifiers based on receptacle nouns, such as *bowl (of)*, which are derived from nouns for concrete physical receptacles, and are used as classifier-like concepts of CONTAINER, CONTENTS/PORTION or MEASURE (Khrizman et al. 2015; Landman 2016). We then characterize what a process of coercion would look like that made use of concepts, such as [bowl of]] and [grain of]], and argue that the differences between these two cases reveals an answer to the accessibility puzzle. Given that our analysis of NPs and counting constructions, which ties together insights from lexical semantics and compositional semantics, requires a slight enrichment of the standard compositional semantic toolbox, we outline how our account is straightforwardly compatible with representations of counting constructions in Appendix A and with VPs in Appendix B.

9.3 Background

9.3.1 Background: Granular Nouns in the Context of Current Mass/Count Theories

Chierchia (2010) argues that count nouns have 'stable atoms' in their denotation. This means that there are entities in their denotation that are atoms in every context (on every admissible precisification of the noun's denotation such that an admissible precisification is an adjustment of the extension of an expression licensed by permissible language use). Mass nouns lack stable atoms in their denotation, that is, for mass nouns, it is a vague matter what the grammatically countable units in noun denotations are: Namely, there is no entity that is an atom in the denotation of the predicate at all contexts (on every admissible precisification of the noun's denotation). In this sense, mass nouns have only unstable individuals in their denotation, and, assuming that counting is counting of stable atoms, mass nouns cannot be directly used in counting constructions. For instance, we have the infelicity of *#three muds*, unless *mud* first undergoes a shift into a plausible contextually determined count interpretation.

Chierchia couches his vagueness-based analysis of the mass/count distinction in supervaluationist terms. How it works is best shown using his paradigm example of a mass noun *rice*. It is vague in the following way. It is not the case that, across all contexts, for example, a few grains or single grains of rice fall under the denotation of the predicate *rice*. But this means that such various quantities of rice are all in the vagueness band of rice; they fall in and out of the denotation of rice are rice atoms. There may be some context c, in which cups of rice are rice atoms. There may also be some c', such that c' extends c, where sums of a few grains are rice atoms. There may also be

some c'' such that c'' extends c', in which single grains are rice atoms. Most importantly, there is, therefore, no entity that is a rice atom at every context of evaluation of *rice*. In this sense, the denotation of *rice* lacks stable atoms, and so is vague. If *rice* has no stable atoms in its denotation, and counting is counting stable atoms, on Chierchia's account, then what *rice* denotes cannot be counted, which motivates its grammatical mass property.

Although rice is Chierchia's paradigm example to motivate his vaguenessbased (supervaluationist) account, granular nouns are in fact problematic for this account. One problem stems from cross- and intralinguistic count/mass variation among granular nouns. For example, *lentil(s)* is count in English, but čočka ('lentil', Czech) is mass. Aware of such data, Chierchia's response is "[w]hat this suggests is that standardized partitions for the relevant substances are more readily available in such languages/dialects" (Chierchia 2010, p. 140). However, were we to accept this explanation for cross-linguistic variation, such a response would still face a challenge in accounting for intralinguistic variation and dual life granular nouns. For example, the German noun Same(n) ('seed(s)'), which is count, has a mass counterpart Saat ('seed') and, as we have seen, some languages have dual life granular nouns such as the English seed. Even if we accept that standardized partitions are more readily available in some languages (for some nouns) than others, it is less plausible to adopt the position that the same language both does and does not make standardized partitions available across two co-extensional lexical items (Saat/Same(n)), let alone for a single lexical item that admits of both a count and a mass sense (seed).

Simply put, one of our worries with Chierchia's account is that it does not seem to capture what we take to be the most puzzling property of granular mass nouns. As observed above, granular mass nouns have entities in their denotation that intuitively count as one, for example, individual grains, seeds and the like. Nevertheless, such 'natural' object units are not accessible to grammatical counting operations, directly or via mass-to-count coercion, even if the context makes them clearly salient and relevant. Chierchia's account, in attempting to reduce the property of being individuated, and hence countable, to the property of having stable atoms does succeed in accounting for what makes certain nouns which denote notionally granular entities grammatically mass, but it does so at the cost of losing any conceptually privileged status for the grain structure that is a core property of the denotations of granular nouns.

One of the key contributions of Landman (2011, 2016) to our understanding of the mass/count distinction is the idea that its motivating property is disjointness. For Landman (2016), count noun concepts specify, relative to context, a disjoint set of entities for counting, their COUNTING BASES. In contrast, the counting bases of mass noun concepts (the set of entities that

could be candidates for counting) are overlapping (not disjoint) and overlap makes 'counting go wrong'.

Rothstein (2010) emphasizes the importance of context for the delimitation of count noun denotations from mass ones, and to this goal coins the term 'counting context'. Count nouns denote sets of entity-context pairs (the entity denoted and the context in which it counts as one), which makes them of type $\langle e \times k, t \rangle$. Mass nouns are of type $\langle e, t \rangle$, that is, they have the standard predicative denotation. This in effect amounts to the claim that the mass/count distinction can be reduced to this typal distinction between mass and count nouns. The main motivating data for the introduction of the counting context into the lexical entries are singular count nouns like *fence, wall* and *rope*, for which, as Zucchi and White (1996, 2001), among others, observed, what counts as 'one' can vary from occasion of use to occasion of use.

One thing that is striking about the mass/count theories of Chierchia's, Landman's and Rothstein's work is that they all integrate some notion of context-sensitivity, albeit each in a slightly different way. Inspired by their proposals in this regard, in Sutton and Filip (2016a), we defended the idea that the mass/count distinction fundamentally relies on the context-sensitive notion of individuation, whereby the relevant context-sensitivity has two sources: one which was inspired by Chierchia's proposal regarding precisification relative to context, and another which was inspired by Landman's and Rothstein's accounts in which context can determine a disjoint set for counting (yielding a count concept) or leave the counting base overlapping (yielding a mass concept). Our main empirical interest in Sutton and Filip (2016a) lay in crossand intralinguistic patterns of count/mass variation. We attempted to motivate the observation that granular (mass/count) nouns, which in English include rice, lentils, beans and collective artifact nouns, including English object mass nouns like *furniture* and the corresponding Dutch count nouns like *meubels*, are distinguished by considerable variation in count/mass lexicalization patterns, cross- and intralinguistically. In contrast, prototypical object denoting nouns (cat, ball) are pretty stably count, and substance denoting nouns (mud, blood) are stably mass, at least in languages with a lexical mass/count distinction. For our analysis of granular nouns, we argued that the way granulars can be individuated is sensitive to context. We proposed that count granulars are interpreted relative to a precisification determined by the context, but that mass granulars are interpreted relative to the intersection of all licensed precisifications such that single grains are excluded from the counting base of the noun. Although our account in Sutton and Filip (2016a) could capture more data than alternative accounts, especially with regard to whether notional classes of nouns will display cross- and intralinguistic mass/count variation, it is, arguably, not as parsimonious as it could be, since it relies on two distinct mechanisms for individuation. In this paper, we will still defend the view that individuation is context-sensitive, and that the semantics of common nouns must reflect this; however, here, we propose that the semantics of the countability of granular nouns and of other notional classes of nouns can be accounted for with a unified account of individuation.

For the account of granular nouns proposed in this paper, of particular interest is the work of Grimm (2012). He enriches the standard mereological semantic toolbox with topological relations so as to be able to articulate, within lexical entries, certain properties which must be taken as basic or unanalyzed in standard mereological accounts. Most importantly for us, Grimm (2012) introduces the notion of CLUSTER, which allows him to differentiate a mereological sum counting as a clustered entity from a mereological sum viewed as one individual entity. For example, the entry for *dog* in (10) (simplified from Grimm (2012)) states that *x* is a realization of the concept *Dog*, and *x* is a *maximally strongly self-connected* (MSSC) individual, i.e. an entity for which every part internally overlaps with the whole.

(10)
$$\llbracket dog \rrbracket = \lambda x [R(x, Dog) \land MSSC(x)]$$

In (11) (simplified from Grimm 2012), the entry for the collective noun *cacwn* ('hornet', Welsh), which refers to a swarm or relatively closely grouped collection of hornets specifies the property $CLUSTER_{P,C}$, which means that *x* is a cluster entity the parts of which share property *P* and are transitively connected under some connection relation *C*. In other words, a set of sum entities that are realizations of swarms of hornets.

(11) $\llbracket \operatorname{cacwn} \rrbracket = \lambda x [R(x, Hornet) \land x \in CLUSTER_{P,C}]$

9.3.2 Formal Background: Extensional Mereology and Frame Semantics

We assume a domain structured as a complete lattice with the bottom element removed, closed under sum \sqcup which is an idempotent, commutative and associative relation. Part \sqsubseteq and proper-part \sqsubset relations are defined as standard:

 $(12) a \sqsubseteq b \leftrightarrow a \sqcup b = b$

(13)
$$a \sqsubset b \leftrightarrow a \sqsubseteq b \land \neg (b \sqsubseteq a)$$

The supremum $\sqcup P$ of a predicate *P* and the upward closure of *P* under sum **P* are also given as standard.

(14) $\Box P =$ the smallest individual *x* such that $\forall y \in P[y \sqsubseteq x]$

$$(15) \qquad ^*P = \{ \sqcup Y : Y \subseteq P \}$$

Two entities a, b overlap $(a \circ b)$:

(16)
$$a \circ b \leftrightarrow \exists x [x \sqsubseteq a \land x \sqsubseteq b]$$

We also make use of the property of predicates being overlapping and disjoint. A predicate is overlapping:

(17)
$$\forall P[OVERLAP(P) \leftrightarrow \exists x \exists y [P(x) \land P(y) \land x \circ y]]$$

(18)
$$\forall P[DISJOINT(P) \leftrightarrow \neg OVERLAP(P)]$$

The formal tool we employ is a form of frame semantics (see Fillmore 1976 for the original proposal). Our version of frame semantics is inspired, in large part, by Type Theory with Records (TTR) (see Cooper 2012 for an introduction an further references), but it is simpler than TTR and stays closer to simply typed semantic theories. The enrichment to frames is, however, necessary. Frames allow us to provide enough detail about lexical information so as to capture subtle differences between the semantics of members of notional noun classes. At the same time, this preserves a standard Montogovian compositional semantics so as to account for compositionality (see Cooper 2012 for details as to why such integration is desirable if not necessary to provide an adequate analysis for at least some natural language data).

Frames, on this approach, are representations of (complex) structured concepts. We assume a basic type f for frames (frames replace propositions), along with other more familiar basic types such as e. Frames are sets of fields. Fields are labeled formulae, with labels to the left of the '=' and formulae to the right. A single frame can have multiple fields (19), and can be recursive in that frames or abstractions over frames can be parts of fields (20). We also allow complex type formation in the usual way. For example, an expression of $\langle e, f \rangle$ can be formed by abstracting over a type e variable within a frame (21).

(19)
$$\begin{bmatrix} l_1 &= \phi_1 \\ \dots &= \dots \\ l_n &= \phi_n \end{bmatrix}$$

(20)
$$\begin{bmatrix} l_1 &= \phi_1 \\ l_2 &= \begin{bmatrix} l_3 &= \phi_2 \\ l_4 &= \phi_3 \end{bmatrix}$$

(21)
$$\lambda x. \begin{bmatrix} l_1 &= \phi_1 \\ l_2 &= \begin{bmatrix} l_3 &= P(x) \\ l_4 &= \phi_2 \end{bmatrix} \end{bmatrix}$$

By specifying labels (l_1) , and paths of labels $(l_i, l_j ...)$ within frames we can select (and then modify) specific parts of frames.⁵ For example, if the expression in (21) is **F**, then the following equivalences hold:

$$\begin{aligned} \mathbf{F}(\mathbf{y}).l_1 &\leftrightarrow \phi_1 \\ \mathbf{F}(\mathbf{y}).l_2 &\leftrightarrow \begin{bmatrix} l_3 &= & P(\mathbf{y}) \\ l_4 &= & \phi_4 \end{bmatrix} \\ \mathbf{F}(\mathbf{y}).l_2.l_3 &\leftrightarrow P(\mathbf{y}) \end{aligned}$$

The move to a frame-based semantics is motivated by our need to represent lexical semantic details in a way that would be, at best, cumbersome for more mainstream formalisms within formal (compositional) semantics. Frames also allow us to represent dependencies between complex semantic structures in a way that would be far more complex to do in more standard formalisms (we give a concrete example below).

We should stress, however, that for any frame, an extensionally equivalent predicate logic expression can be provided. For example, the frame in (22), which is equivalent to the frame in (23), can be 'de-labeled' and converted into the extensionally equivalent propositional logic formula in (24).⁶

(22)
$$\begin{bmatrix} l_1 &= \lambda y. \begin{bmatrix} l_2 &= F(y) \\ l_3 &= G(y) \end{bmatrix} \\ l_4 &= \mathcal{H}(l_1(a).l_2 \\ l_5 &= \mathcal{I}(l_1(b).l_3 \end{bmatrix}$$

(23)
$$\begin{bmatrix} l_1 &= \lambda y. \begin{bmatrix} l_2 &= F(y) \\ l_3 &= G(y) \end{bmatrix} \\ l_4 &= \mathcal{H}(F(a)) \\ l_5 &= \mathcal{I}(G(b)) \end{bmatrix}$$

(24)
$$\mathcal{H}(F(a)) \land \mathcal{I}(G(b))$$

An immediate expressive advantage of these kinds of frames is that one can easily define functions that add or modify information in frames in ways that it is not simple to do with more mainstream formalisms. For example, we can define a function that modifies and adds a further condition on the sub-frame labeled l_1 in (23). The function in (25), applied to the frame in (22), yields the

⁵ The use of paths is also appropriated from TTR (Cooper 2012).

⁶ We do not give the full details of de-labeling and conversion here. However, as a heuristic, we suggest the following: 1. For all fields in a frame, replace any labels within formulae with the formulae that they label and perform any λ reductions. For the frame in (22), this would yield the frame in (23); 2. For all fields with a type *t* formulae in the frame which do not contain a variable in the scope of a λ expression, conjoin the formulae. This yields the formula in (24). In other words, the extension of a frame is characterised by the conjunction of the propositional formulae in its fields.

frame in (26), which is extensionally equivalent to the predicate logic formula in (27).

(25)
$$\lambda \mathcal{F}_{:f} \begin{bmatrix} l_1 &= \mathcal{F}.l_1 \\ l_4 &= \mathcal{F}.l_4 \\ l_5 &= \mathcal{F}.l_5 \\ l_6 &= \mathcal{K}(l_1(c).l_2 \end{bmatrix}$$

(26)
$$\lambda \mathcal{F}_{:f} \begin{bmatrix} l_1 &= \lambda y. \begin{bmatrix} l_2 &= F(y) \\ l_3 &= G(y) \end{bmatrix} \\ l_4 &= \mathcal{H}(l_1(a).l_2) \\ l_5 &= \mathcal{I}(l_1(b).l_3) \\ l_6 &= \mathcal{K}(l_1(c).l_2) \end{bmatrix}$$

(27)
$$\mathcal{H}(F(a)) \wedge \mathcal{I}(G(b)) \wedge \mathcal{K}(F(c))$$

Importantly, without the label-formula (attribute-value) structure of frames, it is not trivial to define a function that could apply to (24) and give the result in (27) without specifying constants such as F within the function \mathcal{F} (which means one would have to define a different function for every argument frame). Frames make simple the representation of modification of rich, structured information in the lexicon of a particular noun while maintaining compositionality, hence we use frames as our representational format.

We use this relatively straightforward compositional mechanism in our nominal semantics. Lexical entries, which represent the basic 'core' meaning of concepts, contain a property of type $\langle e, f \rangle$, where *e* stands for the *entity type* (any concrete object or stuff) and *f* for the *frame type*. Functions apply to this core property and yield a predicate which specifies the counting base for that concept and a predicate which specifies the extension of that concept. These two predicates, as we will go on to argue, differ cross-linguistically, yielding, for example, either a count concept or a mass concept.

Finally, we note that, although notationally different, our adoption of this kind of frame semantics is merely a natural generalization of formalisms already used in state-of-the-art semantics accounts of the mass/count distinction. For example, Landman (2016, 2017) gives the lexical entries of nouns with ordered pairs $\langle body, base \rangle$, which arguably are a simple 'frames' with two fields. Landman uses functions to access each of the projections of these pairs in a manner similar to the way we use labels above. The way in which our approaches differ is that we allow for arbitrarily large numbers of fields and for recursion.

9.4 The Mass/Count Distinction for Granular Nouns

There are three main ingredients for our lexical entries: (i) basic predicates (a part of all lexical entries), (ii) the object identifying function (only a part of

concepts which allow cardinality comparisons in more than constructions) and (iii) schemas of individuation (which are either specific or null). We introduce these three ingredients, and then we show how they interact in lexical entries of 'concrete' nouns that are representative of four different notional classes: granulars, prototypical objects, collective artifacts and substances.

9.4.1 Basic Predicates

A number of different theories of the mass/count distinction assume some kind of basic meaning for nouns. For example, Krifka (1989) assumes numberneutral predicates like *CAT* and *MUD*, which encode the qualitative application conditions for a nominal concept, while Rothstein (2010) assumes a 'root' predicate, which she associates with mass meanings insofar as $P_{\text{root}} = P_{\text{mass}}$. We, too, assume a basic meaning for nominal concepts.

Like Krifka (1989), but unlike Rothstein (2010), we take basic predicates to be number-neutral, rather than notionally mass. Moreover, our adoption of frame semantics (inspired by the work of Fillmore [1975, 1976] and others) enables us to enrich representations of basic predicates with encyclopedic information, including background beliefs/knowledge. At the same time, what stands for us in the foreground is the interface between lexical semantics and compositional semantics, and hence being specific about the parts of the lexical entries that facilitate words to participate in compositional processes (see above, and also, for more details on this type of integration, see Cooper 2012). The mass/count distinction, on our view, turns on properties of counting base predicates, namely whether they specify disjoint or overlapping sets. Counting base predicates are derived from basic predicates via mechanisms of individuation which we will describe in detail below.

Let us illustrate what we mean by basic predicates with two examples: wolf and rice. Take, for example, an expression such as $\lambda x.wolf(x)$, which, following Krifka (1989), is a number-neutral predicate. This, and other such predicates, we propose, can be unpacked into a whole frame which highlights whatever properties (perceptual or essential) there are that specify properties of, in this case, wolves. A suggestion for part of such a frame is given in (28). This frame (along with all other common noun frames, we propose) is separated into three main fields: 'unit', 'collection' and 'extn' (a mnemonic for 'extension'). The unit field specifies a property which includes some background knowledge-based information (e.g. that wolves howl and growl), as well as some mereotopological information (that wolfunits are maximally strongly self-connected (MSSC, Grimm 2012). The 'collection' field specifies information relating to any sums of entities specified in the unit field. The presence of this field in basic predicate frames means that all basic predicates are number-neutral. For *wolf*, this field records information about sums of wolf-units, namely that they typically come in social groups (i.e. packs, represented by $TYP(social_unit(\uparrow (y)))$). The extension field states that the *wolf* concept applies either to individuals or to sums.

(28)
$$\lambda x. \begin{bmatrix} \text{ent.type} = \begin{bmatrix} \text{boundedness} = mssc(y) \\ animacy = animate(y) \end{bmatrix} \\ \text{fur} = \lambda z. \begin{bmatrix} \text{fur} = fur.of(y,z) \\ \text{texture} = fluffy(z) \end{bmatrix} \\ \text{sound} = \begin{bmatrix} \text{sound} = soundof(y) \in \{howl, growl, \ldots\} \end{bmatrix} \\ \dots = \dots \\ \text{collection} = \lambda y. \begin{bmatrix} \text{pack} = * \text{unit}(y) \land TYP(social_unit(\uparrow(y))) \\ \text{behaviour} = territorial(\uparrow(y)) \\ \dots = \dots \\ \text{extension} = \text{unit}(x) \lor \text{collection}(x) \end{bmatrix}$$

A substance denoting predicate, such as *mud*, where 'substance' is understood in the sense of Soja et al. (1991), would not specify any bounded discrete entities at all. This is specified in the unit field, along with other properties like inanimacy, sliminess (when wet) etc.

Object mass nouns which denote artifacts will, despite being mass, contain a 'unit' field. However, we suggest that, because they refer to artifacts, rather than being specified by boundedness conditions such as self-connectedness, units be defined in terms of fulfilling at least one of a bundle of functions that relate to the ability of an item to be used for the purpose or purposes specific to that type of artifact (for example, functions pertaining to furnishing for *furniture*). A similar proposal is made by Grimm and Levin (2017); however, we do not commit to the claim that the specification of these functions must vary cross-linguistically to account for mass/count variation. Crucially, the satisfaction conditions for fulfilling a function will not specify a disjoint set of entities. For example, a vanity fulfills a furnishing function as much as the table, stool and mirror that are its proper parts do. This *overspecification* of what counts of a functional unit (in the sense of Landman 2011) will feed into our theory of individuation as input.

What is notable about granular nouns such as *rice*, *lentil(s)* and *gravel* is that they refer to things that are conceptually individuated objects (i.e. the individual grains like single lentils or bits of gravel), but also typically come in clustered collections. That is to say that most of our interactions with rice, lentils and gravel is with collections of grains etc. Our modest suggestion, which is very much in concord with Grimm's (2012) proposal, is that such properties have a bearing on individuation. Let us partially 'unpack' a predicate such as $\lambda x.lentil(x)$ as an example.

(29)
$$\lambda x. \begin{bmatrix} unit = \lambda y. \\ unit = \lambda y. \\ shape = lens_shaped(y) \\ colour = colour_of(y) \in \{orange, brown, ...\} \\ ... = ... \\ collection = \lambda y. \\ grains = *unit(y) \\ cluster = TYP(cluster)(y) \\ ... = ... \end{bmatrix}$$

The three root fields specified in this frame are, just as with wolf, labeled 'unit', 'collection' and 'extn' (extension). The unit field specifies a property that refers to single grains (including perceptual and inanimacy information). The collection field specifies a property that refers to sum of grains and includes prototypical (but defeasible) mereotopological information, such as typically coming in clusters (TYP(cluster)).⁷ The extn field places an underspecified condition on the entities denoted by the whole frame that they be either a single grain (specified by the unit field) or sums of grains (as specified by the collection field). This typicality of being clustered, we propose, sets granulars aside from other concrete nouns. Pre-theoretically, it also means that granulars have three ways in which one might conceptualize their extension. One way is as single grains, which, plausibly, is how *lentil* in English is conceptualized. Another way is as sums of single grains, which, plausibly is how rice in English is conceptualized. In these two cases, the information that grains come in clusters is understood as a matter of how we typically encounter them. The third and final way is as a clustered entity with hard-wired, as opposed to defeasible, mereotopological restrictions (a granular noun which denotes topologically related clusters of grains, and not, for example, any scattered sums of grains), as is the case for grawn ('grain', Welsh) (Grimm 2012).

We now specify two (families of) functions, which, together, form the machinery for our account of individuation. These are the object identifying function o and context-dependent individuation schemas S_i .

9.4.2 The Object Identifying Function (O)

We have just outlined how predicates, such as $\lambda x.wolf(x)$, $\lambda x.mud(x)$ and $\lambda x.lentil(x)$, which are all of type $\langle e, f \rangle$, can be seen as shorthand for frames which encode bundles of perceptual and/or functional properties. The object identifying function o is of type $\langle ef, ef \rangle$; it applies to such basic frames and

⁷ We assume something akin to Grimm's (2012) account can be applied to spell out the details of predicates like *cluster*.

selects the sub-frame labeled 'unit'. The semantic effect of O is to focus on a sub-lattice of the denotation of the basic frame, namely those entities that count as units. If $\lambda x.P(x)_{:(e,f)}$, then:

(30)
$$\mathcal{O}(\lambda x.P(x)) = \begin{cases} \lambda x.(P(x).\text{unit})(x) & \text{if } P \text{ contains a "unit" field} \\ \lambda x.P(x) & \text{otherwise} \end{cases}$$

In other words, \mathcal{O} is a function that selects, on perceptual or functional grounds, entities that notionally count as 'one' for a given predicate. For example, when \mathcal{O} applies to a frame for $\lambda x.rice(x)$, $\mathcal{O}(rice)$ selects those parts of the frame that specify what counts as 'one', namely single grains. For *cat*, $\mathcal{O}(cat)$ will be a set of single cats. For substance denoting nouns, such as *mud*, we assume that \mathcal{O} is the identity function $\mathcal{O}(mud) = mud$. This captures the fact that substance denoting nouns lack any entities that, on perceptual or functional grounds, notionally count as individuals.

When applied to the frame for *lentil* in (29) above, O(lentil) will return the frame labeled 'unit'. As we stated above, this should pick out the set of single lentils, namely:

(31)
$$\mathcal{O}(\lambda x.lentil(x)) = \lambda x.(lentil(x).unit)(x)$$

$$= \lambda x. \left(\lambda y. \begin{bmatrix} ent_type = \begin{bmatrix} boundedness = mssc(y) \\ animacy = animate(y) \end{bmatrix} \\ shape = lens_shaped(y) \\ colour = colour_of(y) \in \{orange, brown, ...\} \\ ... = ... \end{bmatrix} \right) (x)$$

$$= \lambda x. \begin{bmatrix} ent_type = \begin{bmatrix} boundedness = mssc(x) \\ animacy = animate(x) \end{bmatrix} \\ shape = lens_shaped(y) \\ colour = colour_of(x) \in \{orange, brown, ...\} \end{bmatrix}$$

This set will be disjoint (single lentils are non-overlapping), and thus can form a counting base that is compatible with a grammatical counting operation. In other words, o is a restriction on the *lentil* frame such that it selects the part of the frame that specifies single lentils.

If, on the other hand, a lexical entry only specifies $\lambda x.lentil(x)$ (i.e. the whole frame in (29)) for the entities in the counting base, this will apply to any sums of lentils, an overlapping set. Of course, what overlaps are subsets of individual lentils of a given set of lentils, not one single lentil with another single lentil. Such a set of overlapping subsets would not be compatible with a grammatical counting operation, and so would be the counting base for a mass granular noun that refers to lentils such as the Czech *čočka*.

The object identifying function o alone is insufficient as the basis of an account of individuation which underpins grammatical counting, and by the

same token the grammatical mass/count distinction. This is because the output of applying o to a predicate may or may not yield a disjoint set that is a prerequisite for counting. For example, for *furniture*, the sum entity that is a nest of tables and the individual tables that make up the nest can all count as one unit with respect to *furniture*, but this means that the set of perceptual and or functional units in its denotation is overlapping (see Landman 2011; Sutton and Filip 2016b, 2018). Likewise, for count nouns such as *fence*, the entities that can count as one on perceptual and functional grounds also do not form a disjoint set (fencing around a garden can count as one fence or as many) (Rothstein 2010). Disjoint sets are derived by means of individuation schemas, which, intuitively, represent perspectives on predicates that yields what counts as one in context.

9.4.3 Individuation Schemas

Specific Individuation Schemas S_i : To take the previous example, suppose that a sum entity that is a nest of tables and the individual tables are all functional units in the extension of *furniture* (in the extension of $\mathcal{O}(furniture)$): $\{t_1, t_2, t_3, t_1 \sqcup t_2 \sqcup t_3\} \subseteq \bigcup \mathcal{O}(furniture)$), such that for a formula φ in our representation language, $\bigcup \varphi$ is the extension of φ . Individuation schemas $S_i \in \mathbb{S}$, which are of type $\langle ef, ef \rangle$, apply to a predicate and return a predicate that specifies a maximally disjoint subset of the extension of the argument predicate (Landman 2011):

If $\Downarrow P$ is the extension of *P*, then $\Downarrow (\boldsymbol{S}_i(P)) \subseteq_{\max.disjoint} \Downarrow P$

Applied to the functional units in the extension of O(furniture), in our example above, there will be some S_i and S_k such that:

$$\{t_1, t_2, t_3\} \subseteq \ \ \Downarrow \left(\boldsymbol{\mathcal{S}}_j(\boldsymbol{\mathcal{O}}(furniture)) \right) \text{ and } DISJOINT \left(\Downarrow \left(\boldsymbol{\mathcal{S}}_j(\boldsymbol{\mathcal{O}}(furniture)) \right) \right)$$
$$\{t_1 \sqcup t_2 \sqcup t_3\} \subseteq \ \ \Downarrow \left(\boldsymbol{\mathcal{S}}_k(\boldsymbol{\mathcal{O}}(furniture)) \right) \text{ and } DISJOINT \left(\Downarrow \left(\boldsymbol{\mathcal{S}}_k(\boldsymbol{\mathcal{O}}(furniture)) \right) \right)$$

In other words, different perspectives on furniture lead to different disjoint sets of what is one 'unit' of furniture (see also Landman 2011). The types of \mathcal{O} and \mathcal{S}_i are such that they can be stacked on top of each other, whereby $(\mathcal{S}_i(\mathcal{O}(P)))$ is of type $\langle e, f \rangle$.

For predicates like *cat* that denote individuated objects ('objects' in the sense of Soja et al. 1991) in a stable way across all contexts, sets such as $(S_i(O(cat)))$ will have a stable extension for all $S_i \in S$. Put in the simplest terms, what we view as one cat will not differ from context to next.⁸

⁸ This is, to some extent, an idealisation. In purely extensional terms, there is a certain amount of vagueness, under- or over-specification with respect to what counts as 'one', even for predicates

In contrast, for predicates like *mud* that lack any 'objects' in their denotation, the number of possible maximally disjoint subsets of mud one can form is virtually unrestricted (just think of all of the ways that some mud could be carved up into disjoint clumps).

The Null Individuation Schema S_0 : The schema is inspired by Landman's (2011) idea that, for the denotations of mass nouns, both prototypical like *mud* and object mass nouns like *furniture*, there is in a given context a multiplicity of partitions (his variants) available, none of which, however, is privileged over others as providing 'the' unique individuation schema suitable for counting. What counts as 'one' under one individuation schema overlaps with what counts as 'one' under another. Put differently, mass nouns have overlapping counting bases, which motivates why they cannot be counted, i.e. used in counting constructions (e.g. *#three muds*). In order to capture such observations, we introduce the notion of a NULL INDIVIDUATION SCHEMA, S_0 . When applied to a predicate, the null individuation schema returns the union of the interpretations of that predicate at each of the individuation schemas in S.

(32) $\Downarrow (\boldsymbol{s}_0(P)) = \bigcup_{\boldsymbol{s}_i \in \mathbb{S}} \Downarrow (\boldsymbol{s}_i(P))$

The schema S_0 is *null*, because effectively it amounts to an identity function on *P*. For example, if $\Downarrow P = \{a, b, a \sqcup b\}$, then there are two maximally disjoint subsets of $\Downarrow P$, and hence there are two individuation schemas, S_j , S_k on *P* such that, for example, $\Downarrow (S_j(P)) = \{a, b\}$ and $\Downarrow (S_k(P)) = \{a \sqcup b\}$. Therefore the following holds: $\Downarrow (S_0(P)) = \{a, b\} \cup \{a \sqcup b\} = \{a, b, a \sqcup b\} = \Downarrow (P)$



Figure 9.1

like *cat*. For example, *cat* is vague/over-/underspecifies whether a cat and/or that cat minus its tail count as one. This issue relates to the Problem of the Many (Unger 1980), and we will not address it here.



Figure 9.2

In summary, as Figures 9.1 and 9.2 help to show, applying \mathcal{O} to a basic predicate A potentially restricts the extension of A down to the (perceptually or functionally specified) objects in the extension of A, and the application of an individuation schema S_i restricts this further if $\mathcal{O}(A)$ is not disjoint. However, the application of S_0 leaves any overlap in $\mathcal{O}(A)$ unresolved.

9.4.4 Lexical Entries

We adopt a tripartite structure for lexical entries, following Sutton and Filip (2016b) and Filip and Sutton (2017), and also inspired by some independent suggestions in Landman (2011, 2016). In frame-theoretic terms, this is given as three fields: (i) one labeled 'baspred', which gives the basic predicate; (ii) one labeled 'cbase', which specifies a predicate for the set of entities that is input into the counting function (and is a function on the baspred frame); and (iii) one labeled 'extn', which gives the extension (and is a function on the cbase frame). We give a schema for this in (33). We use (*) to indicate that the upward closure under mereological sum operator is not always present in a lexical entry (it is when the relevant concept denotes pluralities).

(33)
$$\lambda x. \begin{bmatrix} baspred &= \lambda z. P(z) \\ cbase &= \lambda y. \mathcal{S}_i(\mathcal{O}(baspred))(y) \\ extn &= {}^{(*)}cbase(x) \end{bmatrix}$$

The reason this tripartite structure is required is best demonstrated by plurals. For example, for the singular *cat*, the extension and the counting base, namely, the set of entities that count as one, are identical: the set of single cats. For the

plural *cats*, the counting base is still the set of single cats, but the extension is the upward closure of this set under mereological sum.

All ('concrete' common) count nouns have counting base predicates and extension predicates that are indexed to an individuation schema S_i specified by the context of use (see Sutton and Filip 2016b, Filip and Sutton 2017 for justification). This means that under some one particular perspective on individuating the entities involved, there is some disjoint set that is suitable for counting, even if what counts as one may vary from context to context, as in the case of count nouns like *fence* (see also Rothstein 2010). All count nouns are then of type $\langle\langle\langle e, f \rangle, \langle e, f \rangle\rangle$, $\langle e, f \rangle\rangle$, a function from an individuation schema S_i of type $\langle\langle e, f \rangle, \langle e, f \rangle\rangle$ to a function from entities to a frame (of type $\langle e, f \rangle$) that specifies the counting base and the extension. Interpreted in context, and so when an individuation schema is specified, however, this reduces to an expression of type $\langle e, f \rangle$.

Mass nouns, on the other hand, have lexical entries that are saturated with the null individuation schema. This means that they have overlapping counting bases, and hence cannot be counted, i.e. straightforwardly used in grammatical counting operations. All ('concrete') common mass nouns are of type $\langle e, f \rangle$, a function from entities to a frame that specifies the counting base and the extension. The 'slot' in mass noun entries for individuation schemas is filled with the null individuation schema.

Having sketched the most basic assumptions motivating our lexical entries, we will now give examples for prototypical object denoting count nouns (*cat*) and granular count nouns (*lentil*), granular mass nouns (*čočka* 'lentil', mass, Czech), substance mass nouns (*mud*) and 'collective artifact' nouns which include mass nouns (*furniture*) and count-counterparts of object mass nouns (e.g. *huonekalut* 'items of furniture', Finnish).

There are two binary features that can be defined to differentiate these classes. First, [+O]/[-O]: The *cbase* field does/does not contain the object identifying function (\mathcal{O}). Second, [+S]/[-S]: The *cbase* field contains a specific individuation schema (so, [+S]) contains the null individuation schema (so, [-S]). Some of the possible classes, along with natural language exemplars are given in Table 9.1. The generalizations that hold are among the following: Count nouns are all [+O,+S], mass nouns are all [-S]. Where mass nouns are [+O], we expect them to share some properties with other [+O] nouns such as the availability of cardinality comparisons in comparative constructions. (However, see Rothstein, Chapter 8 in this volume, for an in-depth discussion.) For example, if these classifications are right, then we should expect comparative constructions containing both *furniture* and *fencing* ([+O, -S])-like nouns to have a cardinality comparison reading available (as well as a measure reading), but for [-O, -S] mass nouns like *rice* and *mud* to only have a measure reading straightforwardly available in such constructions.

Class	Count/Mass	Example	Categorization
Prototypical object	Count	cat	[+0,+S]
Granular	Count	lentil	[+0,+S]
	Mass	rice	[-0, -S]
Granular Collective artifact	Mass	furniture	[+0, -S]
	Count	huonkalu ^a	[+0,+S]
Substance denoting	Mass	mud	[-0, -S]

Table 9.1 Summary of the semantic categorisation of noun classes

^a '(item of) furniture', Finnish

Prototypical Object Nouns Are [+O,+S] (cat): Nouns denoting prototypical objects, such as *cat*, *chair* or *house* in English, are lexicalized as count nouns, in number marking languages at least. The entry for *cat* in (34) has a cbase (counting base) field that specifies a predicate, $\lambda x.cat(x)$, which is shorthand for a basic predicate expression of type $\langle e, f \rangle$ (a function from entities to a frame that specifies, inter alia, perceptual properties and background knowledge about cats). The object identifying function \mathcal{O} applies to this predicate and returns the set of individual cats. Since the set of single cats is always disjoint, the set of single cats under any schema of individuation, S_i , will be disjoint. This means that our account correctly predicts that nouns like *cat* will be lexicalized as count nouns, since the counting base set is disjoint across all individuation schemas. The extension of *cat* is the same set as the counting base set: the set of single cats under S_i .⁹

(34)
$$[[cat]]^{\boldsymbol{s}_i} = \lambda x. \begin{bmatrix} baspred &= \lambda z.cat(z) \\ cbase &= \lambda y. \boldsymbol{s}_i(\boldsymbol{O}(baspred))(y) \\ extn &= cbase(x) \end{bmatrix}$$

The counting base for the plural noun *cats*, as shown in (35) is the same as for *cat*: the set of single cats under S_i . The extension of *cats* is this set closed under mereological sum: the set of single cats under S_i and sums thereof.

⁹ It is worth bearing in mind that if labels are replaced by the formulae they label, then the expression in (34) is equivalent to the expression in (i). Furthermore, both (34) and (i) are extensionally equivalent to the predicate logic formula in (ii).

(i)
$$\lambda x. \begin{bmatrix} baspred = \lambda z.cat(z) \\ cbase = \lambda y. S_i(\mathcal{O}(cat))(y) \\ extn = S_i(\mathcal{O}(cat))(x) \end{bmatrix}$$

(ii)
$$\lambda x. \boldsymbol{S}_i(\boldsymbol{O}(cat))(x)$$

(35)
$$[[cats]]^{\boldsymbol{s}_i} = \lambda x. \begin{bmatrix} baspred = \lambda z.cat(z) \\ cbase = \lambda y. \boldsymbol{s}_i(\boldsymbol{\mathcal{O}}(baspred))(y) \\ extn = *cbase(x) \end{bmatrix}$$

Count Granular Nouns Are [+O,+S] (lentil): The semantics for a count granular noun such as *lentil* looks very similar to the semantics for nouns like *cat*. As outlined above, the object identifying function \mathcal{O} applies to the base predicate $\lambda x.lentil(x)$ and returns the set of single lentils, a disjoint set. This means that, 'viewed' under any schema of individuation, this set will still be disjoint The extension for *lentil* is the set of single lentils as shown in (36). The extension for *lentils* is the set of single lentils closed under sum (37).

(36)
$$[[lentil]]^{\boldsymbol{s}_{i}} = \lambda x. \begin{bmatrix} baspred = \lambda z.lentil(z) \\ cbase = \lambda y. \boldsymbol{s}_{i}(\boldsymbol{\mathcal{O}}(baspred))(y) \\ extn = cbase(x) \end{bmatrix}$$

(37)
$$[[lentils]]^{\boldsymbol{s}_{i}} = \lambda x. \begin{bmatrix} baspred = \lambda z.lentil(z) \\ cbase = \lambda y. \boldsymbol{s}_{i}(\boldsymbol{\mathcal{O}}(baspred))(y) \\ extn = *cbase(x) \end{bmatrix}$$

Mass Granular Nouns Are [-O,-S] (čočka ['lentil', Czech], rice): Since the semantics for a count granular noun such as *lentil* looks very similar to the semantics for nouns like *cat*, this prompts the question why we should expect there to be any mass granular nouns such as čočka ('lentil', Czech) or rice. Part of our answer lies in the differences there are between frames represented by base predicates such as $\lambda x.lentil(x)$ compared with frames represented by base predicates such as $\lambda x.cat(x)$ (see (28) and (29) above). The frames for base predicates detail perceptual and functional properties, but also mereotopological information, and background and experiential knowledge. For predicates such as $\lambda x.lentil(x)$, this will include the fact that we most frequently encounter granular entities in either aggregated or clustered form. Their referents are formed of grains, but often clustered together such that we cannot even clearly perceive each and every granular entity (see the frame for $\lambda x.lentil(x)$ in (29)). This is not the case for $\lambda x.cat(x)$. As a matter of contingent fact, we usually experience cats as single entities, clearly separated from other cats (even when they are in groups). These contingent facts allow us to more easily 'look past' the granular structure of granular entities and conceptualize entities, such as lentils, as aggregates of grains such that these aggregates lack clear, bounded edges. This is the case for the English *rice*. We clearly know, on the conceptual level, that rice is made up of grains, but we nonetheless do not grammatically individuate rice. The same is also true for mass counterparts of lentils such as the Czech čočka ('lentil', mass), an entry for which is given in (38).

We represent this mismatch between the conceptual level and the level accessible to grammatical counting operations in terms of a distinction between the fields in our lexical entries. The conceptual level is represented by the base predicate $\lambda x.lentil(x)$. The level accessible to grammatical counting is provided in the 'cbase' field. The counting base predicate for *čočka* ('lentil', mass, Czech), $S_0(lentil)$, is equivalent to the frame in (29) under the null individuation schema S_0 . It is this predicate that is the input to grammatical counting operations (see Appendix A); however, this predicate, in the case of mass (granular) nouns, specifies an overlapping set of aggregates of lentils.

(38)
$$[\![\check{c}o\check{c}ka]\!]^{S_i} = \lambda x. \begin{bmatrix} baspred = \lambda z.lentil(z) \\ cbase = \lambda y. S_0(baspred)(y) \\ extn = *cbase(x) \end{bmatrix}$$

Thus, we retain the information that nouns like *rice* and *čočka* denote grains, but also how this individuated structure does not get passed up to the level accessible to grammatical counting.

Substance Denoting Nouns Are [-O, -S] (mud): Substance denoting nouns do not denote objects in the sense of Soja et al. (1991), because there is nothing in their denotation that can be identified reliably as an individual object. Like mass granulars, the lexical entries for nouns in this class do not include the object identifying function $\mathcal{O}_{,.}$ The difference between a noun like mud and a noun like rice is that the former does not refer to objects, even on the notional/conceptual level. However, because substance denoting nouns are [-O, -S] on our analysis, this means that they pattern with mass granular nouns, such as *rice* and *čočka* ('lentil', mass, Czech), on the GRAMMATICAL level. Both mass granular nouns and substance denoting nouns are both bona fide mass nouns, governed by [-S], and neither admits of cardinality comparisons in comparative constructions (see Landman, Chapter 6 this volume), which is governed by [-O].

The cbase field for *mud* is $S_0(mud)$. The predicate *mud* stands for a frame that specifies the perceptual properties of mud (plus some background knowledge, among others). This frame applies to anything with those qualities. The null individuation schema S_0 applies to mud to form a predicate that applies to anything which is mud under any schema of individuation (under any way of dividing mud up into disjoint subsets). The set specified by $S_0(mud)$ is therefore overlapping, and so *mud* is mass. The extension of *mud* is the upward closure of this set under sum.¹⁰

¹⁰ However, since the way in which something like mud can be divided up into disjoint subsets is totally unconstrained by the o function, * $S_0(mud)$ will be coextensional with $S_0(mud)$. This is because any entity that counts as mud could be an entity in a disjoint subset of all mud, therefore the union of all disjoint subsets of mud (* $S_0(mud)$) will be equivalent to the lattice denoted by *mud*.

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(39)
$$\llbracket \text{mud} \rrbracket^{\boldsymbol{S}_i} = \lambda x. \begin{bmatrix} \text{baspred} = \lambda z. mud(z) \\ \text{cbase} = \lambda y. \boldsymbol{S}_0(\text{baspred})(y) \\ \text{extn} = * \text{cbase}(x) \end{bmatrix}$$

Collective Artifact-Denoting Nouns Are [+O,+S] or [+O,-S]: Like granulars, the members of this notional class of nouns display widespread variation in their count/mass lexicalization patterns. For example, the English mass noun *furniture* has a count-counterpart in Finnish *huonekalu(t)* ('piece(s) of furniture', Finnish). Unlike mass granulars, mass collective artifacts have some grammatical properties in common with count nouns insofar as they admit of cardinaility comparisons in 'more than' constructions. We represent this via the inclusion of the object identifying function o in the lexical entries of both count and mass collective artifact nouns. We analyse mass/count variation for this notional class of nous in terms of whether the counting base is interpreted relative to a specific individuation schema S_i .

As we outlined above, the 'unit' field for the furniture concept specifies an overlapping set of any entities that fulfill the functional role of pieces of furniture. Hence O(furniture) will be an overlapping set. This means that under the perspective of an individuation schema S_i , we get a disjoint set (although one whose members may vary with context). Under the null individuation schema S_0 , we get an overlapping set, one which is not fit for counting. Lexical entries of count collective artifact nouns can therefore be described in terms of having the features [+O,+S], and mass collective artifact nouns can be described in terms of having the features [+O,-S]. Mass collective artifact nouns (object mass nouns) are mass (and so [-S]), but do allow for cardinality comparisons in comparative constructions (and so are [+O]).

Thus, for collective artifact nouns, cross- and intralinguitic variation can be accounted for purely via whether the noun encodes an argument for the individuation schema that is salient in the context of utterance (as in (41)), or whether it is saturated with the null individuation schema \mathcal{S}_0 (as in (40)). Hence, we can account for the mass noun *furniture*, which has an overlapping counting base, and the plural count noun *huonekalut* ('items of furniture', Finnish), which, under every individuation schema of utterance, species a disjoint counting base.

(40)
$$\llbracket \text{furniture} \rrbracket^{\boldsymbol{S}_i} = \lambda x. \begin{bmatrix} \text{baspred} = \lambda z. \text{furniture}(z) \\ \text{cbase} = \lambda y. \boldsymbol{S}_o(\boldsymbol{O}(\text{baspred}))(y) \\ \text{extn} = \text{*}\text{cbase}(x) \end{bmatrix}$$

(41)
$$\llbracket \text{huonekalut} \rrbracket^{\boldsymbol{S}_i} = \lambda x. \begin{bmatrix} \text{baspred} = \lambda z. furniture(z) \\ \text{cbase} = \lambda y. \boldsymbol{S}_i(\boldsymbol{\mathcal{O}}(\text{baspred}))(y) \\ \text{extn} = \ast^* \text{cbase}(x) \end{bmatrix}$$

This means that the plural count noun *huonekalut* is semantically related to the mass noun furniture as the substitution of the null individuation schema with the individuation schema specified by the context ([[huonekalut]] $S_i = [[furniture]]^{S_0 \mapsto S_i}$).

9.5 Semantics for Classifier-Like Expressions and Addressing the Accessibility Puzzle

We will now turn to our main accessibility puzzle of why coerced container readings, but not unit readings, are available for mass granular nouns. To this end, we will first give an analysis of container readings and unit readings in measure (pseudo-partitive) constructions with explicit container and unit extracting classifier-like expressions.

9.5.1 Container Classifiers

As pointed out by, among others, Partee and Borschev (2012), Khrizman et al. (2015) and Landman (2016), there at least four different interpretations of measure constructions, such as *two bowls of rice*, which are formed with nouns like *bowl* whose inherent meaning is sortal, namely that of a physical receptacle. Inherently sortal nouns like *bowl* may assume (at least) four relational (classifier-like) meanings when they are used in the measure construction, which we label as follows: (i) a container, (ii) contents, (iii) (free) portion and (iv) measure interpretation. Here we focus on the container reading, that is, two bowls, each of which contains rice. We leave aside measure interpretations on which *two bowls of rice*, for instance, has the mass interpretation of 'rice to the measure of two bowlsful'. Measure interpretations arguably have a different syntactic and semantic structure than container interpretations: The receptacle noun (*bowl*) is interpreted as a measure function which combines with a numeral to form a measure phrase (*three bowls of*) (Rothstein 2011).

On our account, receptacle nouns (*bowl*) in their relational (classifier-like) interpretation that concerns container readings are interpreted as functions from expressions of type $\langle e, f \rangle$ to expressions of type $\langle e, f \rangle$. This means that, for example, *bowls of rice* forms a constituent that is sanctioned in a counting construction such as *three bowls of rice*. This analysis of container readings of pseudo-partitive NPs is in line with Rothstein (2011), Partee and Borschev (2012) and Khrizman et al. (2015), among others.

The entry for the plural sortal noun *bowls* is given in (42).¹¹ We assume a function REL that shifts sortal, receptacle nouns into relational container

¹¹ We assume that this differs from the entry for *bowl* only insofar as it has an extension which is the upward closure of the set of single bowls indicated by * applied to the formular in the *extn* field.

classifiers. Building on some ideas in Rothstein (2011), this function is given in (43). The entry for the container reading of bowl(s) is given in (44).

(42)
$$\llbracket \text{bowls} \rrbracket^{\boldsymbol{S}_i} = \lambda x. \begin{bmatrix} \text{baspred} = \lambda z. bowl(z) \\ \text{cbase} = \lambda y. \boldsymbol{S}_i(\boldsymbol{O}(\text{baspred}))(y) \\ \text{extn} = \text{*}\text{cbase}(x) \end{bmatrix}$$

(43) [[REL]] =

$$\lambda Q_{:\langle e,f \rangle} \cdot \lambda P_{:\langle e,f \rangle} \cdot \lambda x. \begin{bmatrix} baspred &= Q(x) \cdot baspred \\ cbase &= Q(x) \cdot cbase \\ extn &= Q(x) \cdot extn \\ extn_restr &= \forall z. [cbase(z) \land z \sqsubseteq x \\ & \rightarrow \exists v. [P(v) \cdot extn \land contain(z, v)]] \\ precon &= CUM(\lambda y. P \cdot extn(y)) \end{bmatrix} \end{bmatrix}$$

(44)
$$\llbracket \text{bowls of} \rrbracket = \llbracket \text{REL} \rrbracket (\llbracket \text{bowls} \rrbracket^{\mathcal{S}_i}) = \\ \lambda Q_{:\langle e, f \rangle} \cdot \lambda P_{:\langle e, f \rangle} \cdot \lambda x. \begin{bmatrix} \text{baspred} &= \lambda z. bowl(z) \\ \text{cbase} &= \lambda y. \mathcal{S}_i(\mathcal{O}(\text{baspred}))(y) \\ \text{extn} &= \text{``cbase}(x) \\ \text{extn_restr} &= \forall z. [\text{cbase}(z) \land z \sqsubseteq x \\ & \rightarrow \exists v. [P(v). \text{extn} \land contain(z, v)]] \\ \text{precon} &= \text{CUM}(\lambda y. P(y). \text{extn}) \end{bmatrix} \end{bmatrix}$$

The lexical entry in (44) takes as an argument *P* an expression of type $\langle e, f \rangle$, such as $\llbracket apples \rrbracket^{S_i}$ or $\llbracket rice \rrbracket$. It returns an expression of type $\langle e, f \rangle$ which has a counting base predicate that specifies the set of single bowls under any schema S_i . Its extension is this set closed under mereological sum with a restriction that each of the single bowls contains something in the extension of *P*. Finally, precon = CUM($\lambda y.P.extn(y)$) captures the condition, introduced by REL, on the combinatorial properties of container classifiers that nominal terms to which they are applied have a cumulative denotation (Krifka 1998):

(45)
$$\operatorname{CUM}(\mathbf{P}) \leftrightarrow \forall x \forall y [(P(x) \land P(y)) \to P(x \sqcup y)]$$

This condition is straightforwardly satisfied by bare mass and plural terms (*a bowl of rice/apples*), but not by singular count terms (#*a bowl of (an) apple)*).

The result of combining the function in (44) with $[apples]^{s_i}$ is given in (46) (with labels replaced with full formulas to aid readablity).

(46)
$$\begin{bmatrix} bowls of apples \end{bmatrix} = \begin{bmatrix} bowls of \end{bmatrix} (\llbracket apples \end{bmatrix}^{\mathcal{S}_i}) = \\ \lambda x. \begin{bmatrix} baspred &= \lambda z. bowl(z) \\ cbase &= \lambda y. \mathcal{S}_i(\mathcal{O}(bowl))(y) \\ extn &= *\mathcal{S}_i(\mathcal{O}(bowl))(x) \\ extn_restr &= \forall z. [\mathcal{S}_i(\mathcal{O}(bowl))(z) \land z \sqsubseteq x \\ & \rightarrow \exists v. [*\mathcal{S}_i(\mathcal{O}(apple))(v) \land contain(z, v)]] \\ precon &= CUM(\lambda y^* \mathcal{S}_i(\mathcal{O}(apple))(y)) \end{bmatrix} \end{bmatrix}$$

This yields a set of entities that are single bowls or sums thereof, each of which contains apples such that counting proceeds in terms of how many such bowls there are. The expression in (46) is extensionally equivalent to the predicate logic formula in (47):

(47)
$$\lambda x.[*S_i(\mathcal{O}(bowl))(x) \land \forall z.[S_i(\mathcal{O}(bowl))(x) \land z \sqsubseteq x \rightarrow \exists v.[*S_i(\mathcal{O}(apple))(v) \land contain(z,v)] \land \text{CUM}(\lambda y^*S_i(\mathcal{O}(apple))(y))]$$

The result of combining the function in (44) with rice is given in (48) (with labels replaced with full formulas).

(48)
$$\begin{bmatrix} \text{bowls of rice} \end{bmatrix} = \begin{bmatrix} \text{bowls of(rice)} \end{bmatrix} = \\ \lambda x. \begin{bmatrix} \text{baspred} &= \lambda z. bowl(z) \\ \text{cbase} &= \lambda y. \mathcal{S}_i(\mathcal{O}(\text{bowl}))(y) \\ \text{extn} &= {}^*\mathcal{S}_i(\mathcal{O}(\text{bowl}))(x) \\ \text{extn_restr} &= \forall z. [\mathcal{S}_i(\mathcal{O}(\text{bowl})) \land z \sqsubseteq x \\ & \rightarrow \exists v. [{}^*\mathcal{S}_0(\mathcal{O}(rice))(v) \land contain(z, v)]] \\ \text{precon} &= CUM(\lambda y^* \mathcal{S}_0(\mathcal{O}(rice))(y)) \end{cases}$$

This yields a set of entities that are single bowls or sums thereof, each of which contains rice such that counting proceeds in terms of how many such bowls there are.

9.5.2 Unit Extracting Classifiers

The intuitive idea that underlies the semantics of unit extracting classifiers, such as *grain of*, is that they 'zoom in' on and make accessible the units that are inherent in a granular mass noun's denotation, that is, more precisely, units specified in its basic predicate frame. The result is something extensionally equivalent to a count granular expression. Formally, this is achieved by two elements encoded by unit extractors, such as *grain of*: the object identifying O function which applies to the basic predicate frame and identifies any entities that can count as single grains; and the individuation schema of utterance S_i which may select a subset of these entities as those individuated in the context.¹²

The lexical entry for the unit extracting classifier grain of is given in (49). It applies to an expression P of type $\langle e, f \rangle$ such as rice and returns a set of entities or sums thereof that are identified as objects in the extension of P (via the object identifying function O) that count as individuated under the individuation schema of utterance S_i . The set of single entities are specified as the counting base.

¹² For example, in relatively rare cases where two grains have grown such that they intermingle with one another, the individuation schema will determine whether it counts as one or two grains.

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(49)
$$\llbracket \text{grains of} \rrbracket = \lambda P \cdot \lambda s \cdot \lambda x. \begin{bmatrix} \text{baspred} = P(x) \cdot \text{baspred} \\ \text{cbase} = \lambda y \cdot s(\mathcal{O}(P(x) \cdot \text{baspred}))(y) \\ \text{extn} = \text{*} \text{cbase}(x) \end{bmatrix}$$

The result of applying *rice* to *grain of* is given in (50). This yields the set of single grains of rice and sums thereof (under schema S_i) such that the single grains of rice (under schema S_i) count as one.

(50)
$$\begin{bmatrix} \text{grains of rice} \end{bmatrix}^{\boldsymbol{S}_i} = \lambda s. \lambda x. \begin{bmatrix} \text{baspred} = \lambda z. rice(z) \\ \text{cbase} = \lambda y. s(\boldsymbol{O}(\text{baspred}))(y) \\ \text{extn} = \text{*cbase}(x) \end{bmatrix}$$
$$= \lambda x. \begin{bmatrix} \text{baspred} = \lambda z. rice(z) \\ \text{cbase} = \lambda y. \boldsymbol{S}_i(\boldsymbol{O}(\text{baspred}))(y) \\ \text{extn} = \text{*cbase}(x) \end{bmatrix}$$

9.5.3 Addressing the Accessibility Puzzle

Now that we have given an account of the semantics of granular nouns (Section 9.4) and of container and unit extracting expressions (Section 9.5.2), we are in a position to address the accessibility puzzle. Recall that what we dub the accessibility puzzle is as follows, repeated here for convenience:

Why should conceptually and perceptually salient object units in the denotations of object mass nouns and granular mass nouns not be *directly* accessible by semantic counting operations, nor facilitate the mass-to-count *coercion*?

Specifically, when it comes to granular mass nouns the twin data to be explained are:

- **Implicit unit extracting classifiers:** Counting constructions, such as *three rices*, CANNOT be coerced into unit interpretations, such as *three GRAINS OF rice*, whereby the relevant units for counting are inherent in the meaning of *rice*.
- **Implicit container classifiers:** Counting constructions, such as *three rices*, can be coerced into portion readings, e.g. *three BOWLS OF rice*, whereby the relevant portions are recovered from context.

The account of individuation we have presented here has two key mechanisms that work in unison with each other: the object identifying function \mathcal{O} , which identifies objects which are possible candidates for being individuated, and the schema of individuation S_i , which selects some subset of these as entities to be counted in the context (see Figure 9.1).

We propose to derive the restrictions on coercion by using the semantics of of pseudo-partitive constructions (*two bowls of rice*) and unit extracting

constructions (*two grains of rice*) given above as a window on coercion. In particular, we propose that a difference in the semantics of relational (classifier-like) concepts, such as [bowl of], and unit extracting concepts, such as [grain of], can be used to explain why the former can be used as an implicitly provided functor to repair a type clash, whereas the latter cannot.

Coerced mass-to-count shifts require the addition of the CONTAINER concept retrieved from the context which supplies the unit for counting (both the object identification function and the individuation schema in our terms). These kinds of mass-to-count shifts do not exploit any discrete units which are inherent in the lexical structure of nouns, and a part of their core meaning. So when counting bowls of rice, as in *three bowls of rice* under its CON-TAINER interpretation, the individuation criterion for counting comes from the contextually determined concept [[bowl]] that is supplied externally to the noun *rice* in order to resolve the type clash triggered by *two* when it directly combines with *rices* and so restores compositionality.

In contrast, when counting grains of rice, the semantics of expressions such as *grain of* is to make available to the grammar those entities that we know of as natural grain-units as part of our knowledge of rice. On our account, this is done via the introduction of the object identifying function \mathcal{O} and the schema of individuation \mathcal{S}_i into the *cbase* field of the [[rice]] frame. This creates a predicate that specifies a disjoint counting base. On the level of the grammar, this has the effect that unit extracting classifiers encode a shifting operation from mass to count: [$_{CL}$ grain of] [$_N$ rice] \Rightarrow [$_N$ grain of rice]_{count}.

In summary, when counting bowls of rice, individuation turns on individuating bowls and their contents. When counting grains of rice, individuation turns on making entities (the grains) that are anyway specified as part of the *rice* concept available to the grammatical counting operation. With this distinction in hand, we may now precisely say how it has an impact on restricting mass-to-count coercion.

9.5.4 Implicit Unit Extracting Classifiers

Explicit unit extracting expressions, on our analysis, operate by introducing the object unit function and a schema of individuation into the counting base field of the argument mass noun such that the mass noun concept shifts from lacking disjoint individuation criteria (being mass) to having disjoint individuation criteria (being count). *Implicit* unit-extractor concepts would have to perform the same task. In other words, there would have to be some implicit concept available whose sole task would be to shift mass nouns into count nouns, i.e. some general function that shifts [-O, -S] concepts to [+O, +S] concepts (i.e. the free insertion of the tools for individuation (0 and S_i) into

any mass concept). Crucially, this would apply not only to mass granular nouns like *rice* but also substance denoting nouns like *mud*.

It is this general function that shifts [-O, -S] concepts to [+O, +S]concepts that, we propose, is blocked as a coercion mechanism for any language with a grammaticized lexical mass/count distinction. Here is why. Suppose that a language L has lexicalized mass/count distinction. Further suppose that a mass-to-count unit shift was available in L for mass granular nouns. This mass-to-count unit shift, we have argued, would be equivalent to a generalized mass-to-count shift (that could apply to any mass noun in a suitable context). Hence, L would have available a generalized mass-tocount shift. However, the availability of a generalized mass-to-count shifting operation is incompatible with L being a language with a lexicalized mass/count distinction (since any mass noun could be used as a count noun given some salient, disjoint set of entities in the context). Hence, it cannot be the case that L has a mass-to-count unit shift for mass granular nouns if L is a language with a lexicalized mass/count distinction. In other words, a granular unit-shifting function is equivalent to a general function that shifts [-O, -S] concepts to [+O, +S] concepts, and licensing this shift in a language is incompatible with a lexicalized mass/count distinction in that language. Indeed, arguably, Yudja is a language which has such a function and also lacks a lexicalized mass/count distinction (see Lima 2014 for arguments for the latter claim.)

9.5.5 Implicit Container Classifiers

Explicit container classifiers, such as *bowl*, contribute the individuation criteria of the receptacle concept (e.g. $\lambda x. S_i(O(bowl))(x)$) and further require that the receptacle contains things/stuff that are in the extension of the argument nominal concept. That means that, unlike explicit unit extracting classifiers (*grain of*), container classifiers do not simply encode a function that inserts the object unit function O and the relevant contextual schema of individuation S_i into the frame for the common noun to which they apply.

The fact that unit extracting and container classifier concepts work in these distinct ways has an impact on whether or not they can be retrieved from the context and used to repair a type clash between, for example, a count quantifier or numerical expression and a mass noun. We have just argued that languages with a grammaticized, lexical mass/count distinction cannot have a mass-to-count unit-shifting operation that shifts granular mass noun concepts into count noun concepts by making available the natural granular units to the grammatical counting operation. This means that no implicitly provided concept like $[[grain of]]^{S_i}$ can be used to resolve a type clash between, for example, [[three]] and [[rice(s)]]. Crucially, our argument for this turned on the fact that a granular

unit-shifting operation would be equivalent to a generalized mass-to-count shifting operation.

The story with implicitly provided container classifier concepts is different. If, for example, [[bowl of]] S_i is salient as a container concept in the context, it can be used to resolve a type clash between, for example, [[three]] and [[rice(s)]], since the [[$_{CL}$ bowl of]] S_i shift is not equivalent to a generalized shifting operation from mass noun concepts to the equivalent count noun concept. This is for at least two reasons: (i) [[$_{CL}$ bowl of]] S_i adds a concept with its own individuation criteria ($\lambda x. S_i(\mathcal{O}(bowl))(x)$) and so does not modify the individuation criteria of [[rice]]; (ii) [[$_{CL}$ bowl of]] S_i is not primarily a mass-to-count shifting operation – it applies to concepts as well as mass concepts.

Since container classifier concepts such as $[bowl of]^{S_i}$ are not equivalent to a generalized shifting operation from mass noun concepts to the relevant count noun concept, there is no reason why they should not be, modulo context, employed to resolve a count/mass type clash. In other words, if suitable, salient-in-the-context receptacle concepts are available, expressions like *three rices* can be used to mean things like *three BOWLS OF rice* without amounting to licensing a shift that is incompatible with the relevant language having a lexicalized grammatical mass/count distinction.

9.5.6 The Importance of Granular Nouns in Mass/Count Theories

If our analysis is on the right track, then, intriguingly, it opens up the possibility of treating granular nouns and explicit unit extracting classifiers as testing grounds for the mass/count distinction in a way similar to the role assigned by Chierchia (2010) to object mass nouns:

What makes fake mass nouns interesting is that they constitute a fairly recurrent type of non-canonical mass nouns, and yet they are subject to micro-variation among closely related languages. For all we know, the phenomenon of fake mass appears to be restricted to number marking languages. It is unclear that classifier languages like Mandarin and number-neutral languages like Dëne display a class of cognitively count nouns with the morphosyntax of mass nouns. In view of this intricate behaviour, fake mass nouns arguably constitute a good testing ground for theories of the mass/ count distinction. (Chierchia 2010, p. 111)

First notice that we can replace 'fake mass' in the above quote with 'granular mass' and make a parallel point. Extrapolating further, we can put forward a hypothesis regarding the available mass-to-count shifts in particular types of languages: If a language has a grammaticized lexical mass/count distinction as is typical with number marking languages such as English, German and Finnish, and has expressions equivalent to *grain (of)*, it cannot license implicit mass-to-count unit-shifting operations.

Classifier languages have been argued to reflect only the object/substance distinction in their lexical nominal system (with countability reflected in the classifier system). On the assumption that the semantics of individuating classifiers is to make uncountable concepts (such as kinds) into countable predicates, classifier languages also cannot permit coerced mass-to-count unit shifts since this, too, given our analysis, would be equivalent to a generalized mass-to-count shifting operation.

If a language has a relatively impoverished classifier (or classifier-like) system, and generally lacks other reflexes of a lexicalized mass/count distinction (such as number marking and specialized mass quantifiers), then there is no in-principle reason why there should not be a generally licensed means of grammatically counting the grains in the denotations of granular nouns. However, given this, not only would such a language lack mass granular nouns, it should also license the grammatical counting of all nouns (including substance nouns, given a suitable context). One language that potentially meets these criteria is Yudja. Yudja has only a few classifier-like expressions that are highly restricted in their distribution (Lima 2019), and all notionally mass nouns are countable in Yudja modulo a suitable context (Lima 2014).

9.6 Conclusions and Comparisons

We have provided a detailed account of the lexical semantics of granular (mass/count) nouns couched within a broader theory of the semantics of lexical nouns and the mass/count distinction. We have also given analyses of measure (pseudo-partitive) constructions formed with what are inherently sortal receptacle Ns, such as *bowl*, under their container (classifier-like) interpretation and with unit extracting classifier expressions like *grain of*.

We have defended an account of countability based on the interaction of three key ingredients: (i) more detailed lexical semantic representations of basic predicates than has been proposed by previous algebraic, mereo(topo) logical analyses of the mass/count distinction so far (which required a more expressive representational format, namely frames); (ii) the object identifying function \mathcal{O} and (iii) schemas of individuation $S_i \in S$ in particular contexts and the null individuation schema S_0 . For example, we can account for why part of the concept for mass granulars, such as *rice*, is that they come in grains while still not being accessible to grammatical counting operations. Although part of the basic predicates for nouns like *rice* specify a denotation made up of single grains, these single grains are not uniquely specified by the part of the lexicon that is accessed by grammatical counting operations; in our terms, they are not specified in the cbase field of lexical entries for granular nouns.

Arguably, our analysis is an improvement over Chierchia's (2010) theory in which, essentially, the theoretical functions of o and s are merged into one

supervaluationist theory, the result of which, we argued, was to lose the ability to maintain a conceptually privileged place for the granular nature of granular nouns. This is because, according to Chierchia (2010), mass granular nouns have unstable entities at the 'bottom' of their denotations, just like (mass) substance nouns do. In contrast, our frame-based representation can record the fact that mass granular nouns refer to stuff made up of grains, while also encoding why these entities are not available to grammatical counting operations.

One open question is whether our answer to the accessibility puzzle could, in principle, be adopted by (modifying) other theories of the mass/count distinction. The crucial ingredients such a theory would need to have are:

- (i) a distinction between the semantics of container classifier-like expressions such as *bowl of* and unit extracting expressions such as *grain of* such that the latter function to make the units inherent in our general knowledge pertaining to a nominal concept available for counting. Without this feature, one cannot use our explanation for why *three rices* can mean 'three bowls of rice', but not 'three grains of rice'.
- (ii) a distinction between a function that determines the set of possible entities for counting (our o), and one that identifies the subset of those for counting in a particular context (our s_i).

This distinction enables one to explain why mass granular nouns pattern grammatically with substance mass nouns but many languages also have count granular nouns. Taken together, (i) and (ii) also facilitate an explanation of the semantics of unit extracting expressions. For example, if a theory were to encode only the equivalent of/an alternative for our S_i and lack the equivalent of our O (as many other context-sensitive theories of the mass/count distinction seem to do, among them Rothstein 2010 and Chierchia 2010), and that theory were to abide by condition (i), then there would be no principled way to restrict the equivalent of S_i in that theory to access only the grains in granular nouns, since with only the equivalent of S_i and with no equivalent of O sums of grains would, in principle, be accessible to grammatical counting.

Numerical expressions, when used adjectivally as predicate modifiers, are functions of type $\langle ef, ef \rangle$ on our account. Applied to the entry for a count noun such as $[cats]]^{S_i}$, they return a function from entities to frames that has the same cbase and extn fields as the property that is the argument of the type $\langle ef, ef \rangle$ function, but adds a restriction to the extension such that it is a set of sum entities that have a cardinality of *n* with respect to the counting base property. The lexical entry for *three* is given in (A1), and the result of composing this with $[cats]]^{S_i}$ is given in (A2):

(A1)
$$\llbracket \text{three} \rrbracket = \lambda P_{:\langle e,f \rangle} \cdot \lambda x. \begin{bmatrix} \text{baspred} &= P(x) \cdot \text{baspred} \\ \text{cbase} &= P(x) \cdot \text{cbase} \\ \text{extn} &= P(x) \cdot \text{cbase} \\ \text{extn}_{-} \text{restr} &= P(x) \cdot \text{extn} \\ \text{extn}_{-} \text{restr} &= \mu_{card}(x, \text{cbase}, 3) \end{bmatrix}$$

(A2)
$$\llbracket \text{three cats} \rrbracket^{\mathcal{S}_i} = \llbracket \text{three} \rrbracket (\llbracket \text{cats} \rrbracket^{\mathcal{S}_i}) = \lambda x. \begin{bmatrix} \text{baspred} &= \lambda z. cat(z) \\ \text{cbase} &= \lambda y. \mathcal{S}_i(\mathcal{O}(\text{baspred}))(y) \\ \text{extn} &= \text{*} \text{cbase}(x) \\ \text{extn}_{-} \text{restr} &= \mu_{card}(x, \text{cbase}, 3) \end{bmatrix}$$

We assume, following Landman (2011, 2016), that the cardinality function μ_{card} is defined only for properties that specify disjoint sets of individuals. This is what blocks counting of mass nouns which, by assumption, have ovelapping (non-disjoint) counting base sets.

An extensionally equivalent proposition to (A2) in predicate logic is given in (A3).

(A3)
$$\lambda x.^* \boldsymbol{S}_i(\boldsymbol{O}(cat))(x) \wedge \mu_{card}(x, \lambda y. \boldsymbol{S}_i(\boldsymbol{O}(cat))(y), 3)$$

In words, this is the set of sums of cat units under individuation schema S_i that have a cardinality of 3 with respect to the predicate for single cat units under individuation schema S_i .

Given our proposal of a frame-based semantics for NPs including numerical NPs, we should provide some indication of how our NP semantics could combine with a VP. There are, inevitably, many issues that we will not even begin to address in this paper (cumulativity, distributivity and quantifier scope, to name but a few), and it is only within the scope of this paper to provide an outline of how such compositional mechanisms could work.

We will adopt a fairly traditional neo-Davidsonian approach with some insights from Partee (1986). Intransitive VPs such as *play* will be of type $\langle e, \langle v, f \rangle \rangle$ as opposed to the standard $\langle e, \langle v, t \rangle \rangle$ (with v as the type for eventualities). We also adopt a version of Partee's (1986) A shift, which was originally proposed as type $\langle \langle et \rangle, \langle \langle et \rangle, t \rangle \rangle$ (i.e. shifting a predicate to a GQ). Here, since we assume a frame-based neo-Davidsonian semantics, our A shift is $\langle \langle ef \rangle, \langle \langle e, \langle v, f \rangle \rangle$.

Let us take *three cats play* as an example. The frame semantics for *play* will just be a frame-based interpretation of a standard neo-Davidsonian representation:

(B1)
$$\llbracket play \rrbracket = \lambda x \cdot \lambda e. \begin{bmatrix} extn = play(e) \\ agent = agent(e, x) \end{bmatrix}$$

In oder to compose with an expression of type $\langle e, f \rangle$ such as [[three cats]] \mathcal{S}_i in (A2) above, we assume a type shifting operation that converts a predicate NP into a GQ. We call this A_f for the frame-based version of Partee's A shift, the semantics for which we give in (B2).

(B2)
$$A_f = \lambda P_{:\langle e,f \rangle} \cdot \lambda V_{:\langle e,\langle v,f \rangle\rangle} \cdot \lambda e \cdot \exists x \cdot \begin{bmatrix} extn = V(e)(x) \cdot extn \\ agent = V(e)(x) \cdot agent \\ agent_restr = P(x) \end{bmatrix}$$

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This gives us the 'vanilla' derivation for three cats play:

(B3)
$$\| \text{three cats play} \|^{S_i} = \| A_f \| (\| \text{three cats} \|^{S_i}) (\| \text{play} \|)$$

$$= \lambda V.\lambda e. \exists x. \begin{bmatrix} \exp n &= V(e)(x). \exp n \\ \operatorname{agent_restr} &= V(e)(x). \operatorname{agent} \\ \operatorname{baspred} &= \lambda y. \mathcal{S}_i(\mathcal{O}(\operatorname{baspred}))(y) \\ \operatorname{extn} &= * \operatorname{cbase}(x) \\ \operatorname{extn_restr} &= \mu_{card}(x, \operatorname{cbase}, 3) \end{bmatrix} \right] \quad (\llbracket \text{play} \|)$$

$$= \lambda e. \exists x. \begin{bmatrix} \exp n &= play(e) \\ \operatorname{agent_restr} &= \begin{bmatrix} \operatorname{baspred} &= \lambda z. \operatorname{cat}(z) \\ \operatorname{extn_restr} &= \mu_{card}(x, \operatorname{cbase}, 3) \end{bmatrix} \end{bmatrix}$$

$$= \lambda e. \exists x. \begin{bmatrix} \exp n &= play(e) \\ \operatorname{agent_restr} &= \begin{bmatrix} \operatorname{baspred} &= \lambda z. \operatorname{cat}(z) \\ \operatorname{cbase} &= \lambda y. \mathcal{S}_i(\mathcal{O}(\operatorname{baspred}))(y) \\ \operatorname{ctn} &= * \operatorname{cbase}(x) \\ \operatorname{extn_restr} &= \mu_{card}(x, \operatorname{cbase}, 3) \end{bmatrix} \end{bmatrix}$$

Existential closure can then yield an expression of type f which yields an expression extensionally equivalent to the event semantics formula in (B4):

(B4)
$$\exists e.\exists x.[play(e) \land agent(e,x) \land^* S_i(\mathcal{O}(cat))(x) \land \mu card(x, \lambda y. S_i(\mathcal{O}(cat))(y), 3)]$$

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